Quiz 6

Your Name:
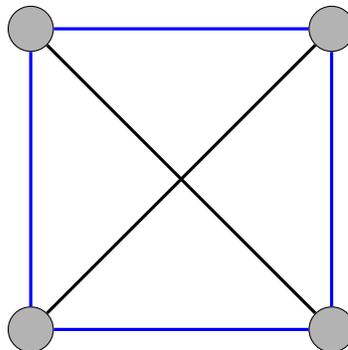
Instructions

This quiz consists of two parts. In each part complete **two** problems for a total of four problems. You should provide detailed solutions on your own paper to the problems you choose to complete. I expect your solutions to contain sufficient justification. I also expect your solutions to be *well-written, neat, and organized.* Incomplete thoughts, arguments missing details, and scattered symbols and calculations are not sufficient. Each problem is worth 4 points for a total of 16 points. Good luck and have fun!

Part A

Complete **two** of the following problems.

A1. Find all the ways to arrange four points in the plane so that only two distances occur between any two points. Below is one possibility. Find the remaining configurations.

A2. Consider an equilateral triangle with side lengths of 2 units. Find an arrangement of 5 distinct points or argue that no such arrangement exists such that all 5 points are in the interior of the triangle and every pair of points is at least 1 unit apart.

A3. The first vote counts of the papal conclave resulted in 33 votes each for candidates A and B and 34 votes for candidate C. The cardinals then discussed the candidates in pairs. In the second round each pair of cardinals with differing first votes changed their votes to the third candidate they did not vote for in the first round. The new vote counts were 16, 17 and 67. They were about to start the smoke signal when Cardinal Ordinal shouted "wait". What was his reason?

Part B

Complete **two** of the following problems.

B1. A week before Thanksgiving, a sly turkey is hiding from a family that wants to cook it for the holiday dinner. There are five boxes in a row (numbered 1, 2, 3, 4, 5), and the turkey hides in one of these boxes. Each night, the turkey moves one box to the left or right, hiding in an adjacent box the next day. Each morning, the family can look in one box to try to find the turkey. How can the family guarantee they will find the turkey before Thanksgiving dinner? *Hint:* See if you can first sort out the case when the turkey is in an even-numbered box on the first day.

B2. Suppose you have 9 coins, all identical in appearance and weight except for one that is either heavier or lighter than the other 8 coins. What is the minimum number of weighings one must do with a two-pan scale in order to identify the counterfeit and determine its weight? Justify that your answer is correct by describing an algorithm.

B3. Consider the following dialogue.

> William: I have three children.
>
> Harry: What are their ages?
>
> William: The product of their ages is 36.
>
> Harry: I still don't know their ages.
>
> William: The sum of their ages is your apartment number.
>
> Harry: I still don't know their ages!
>
> William: The oldest plays football.
>
> Harry: Now I know their ages.

What are the ages of William's children?