Quiz 7

Your Name:

Instructions

This quiz consists of two parts. In each part complete **two** problems for a total of four problems. You should provide detailed solutions on your own paper to the problems you choose to complete. I expect your solutions to contain sufficient justification. I also expect your solutions to be *well-written, neat, and organized*. Incomplete thoughts, arguments missing details, and scattered symbols and calculations are not sufficient. Each problem is worth 4 points for a total of 16 points. Good luck and have fun!

Part A

Complete **two** of the following problems.

A1. My Uncle Robert owns a stable with 25 race horses. He wants to know which three are the fastest. He owns a race track that can accommodate five horses at a time. What is the minimum number of races required to determine the fastest three horses?

A2. Show that in any set of seven different positive integers there are three numbers such that the greatest common divisor of any two of them leaves the same remainder when divided by three.

A3. In the senate of the Klingon home world no senator has more than three enemies. Show that the senate can be separated into two houses so that nobody has more than one enemy in the same house.

Part B

Complete **two** of the following problems.

B1. You need to pack several items into your shopping bag without squashing anything. The items are to be placed one on top of the other. Each item has a weight and a strength, defined as the maximum weight that can be placed above that item without it being squashed. A packing order is safe if no item in the bag is squashed, that is, if, for each item, that item's strength is at least the combined weight of what's placed above that item. For example, here are three items and a packing order:

Ordering	Item	Weight	Strength
Top	Apples	5	6
Middle	Bread	4	4
Bottom	Carrots	12	9

This packing is not safe. The bread is squashed because the weight above it, 5, is greater than its strength, 4. Find all safe orderings of the three items above or argue that there are none.

Written by D.C. Ernst

B2. Two prisoners are locked away in two separate towers, say North Tower and South Tower, and each tower has its own prison guard. Each morning, the respective guards toss a fair coin and then radio the guard in the other tower and report the outcome (heads or tails) of their coin toss. The guard then shows the prisoner in his/her respective tower the outcome of the coin toss in the opposite tower. At this point, each prisoner must guess the outcome of the coin toss that occurred in his/her tower. If at least one of the prisoners guesses correctly, then the prisoners survive another day. If both guess incorrectly, then both will be executed. Is there a strategy that the prisoners can implement that will ensure their survival (until they die of old age in prison) or are they doomed to eventually guess incorrectly and perish? You may assume that prior to being permanently locked up, the prisoners had a few minutes to concoct a plan.

B3. The figure below shows an equilateral triangle ABC with an inscribed semicircle of radius R that is tangent to sides AB and AC, and inscribed circle of radius r that is tangent to the triangle and the semicircle. Find the value of r/R.