Quiz 1

Your Name:

Instructions

This quiz consists of two parts. In each part complete **two** problems for a total of four problems. You should provide detailed solutions on your own paper to the problems you choose to complete. I expect your solutions to contain sufficient justification. I also expect your solutions to be *well-written, neat, and organized*. Incomplete thoughts, arguments missing details, and scattered symbols and calculations are not sufficient. Each problem is worth 4 points for a total of 16 points. Good luck and have fun!

Part A

Complete **two** of the following problems.

A1. Imagine a hallway with 1000 doors numbered consecutively 1 through 1000. Suppose all of the doors are closed to start with. Then some dude with nothing better to do walks down the hallway and opens all of the doors. Because the dude is still bored, he decides to close every other door starting with door number 2. Then he walks down the hall and changes (i.e., if open, he closes it; if closed, he opens it) every third door starting with door 3. Then he walks down the hall and changes every fourth door starting with door 4. He continues this way, making a total of 1000 passes down the hallway, so that on the 1000th pass, he changes door 1000. At the end of this process, which doors are open and which doors are closed? You must justify your answer.

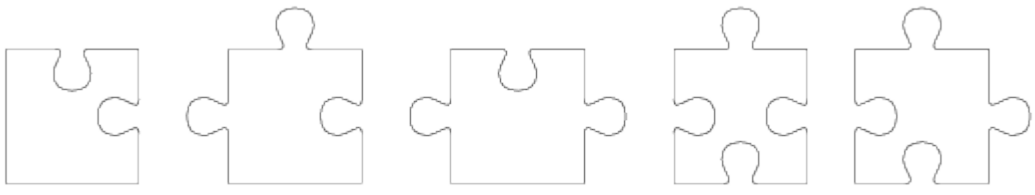
A2. Imagine you have n^2 pebbles, each occupying one square on a $n \times n$ chess board. Suppose that each pebble must move to an adjacent square by only moving up, down, left, or right. For what values of n is the puzzle solvable? For what values of n is the puzzle unsolvable? Justify your answers by either providing a method for a solution or an explanation for why a solution is not possible.

A3. I have 10 sticks in my bag. The length of each stick is an integer. No matter which 3 sticks I try to use, I cannot make a triangle out of those sticks. What is the minimum length of the longest stick?

Part B

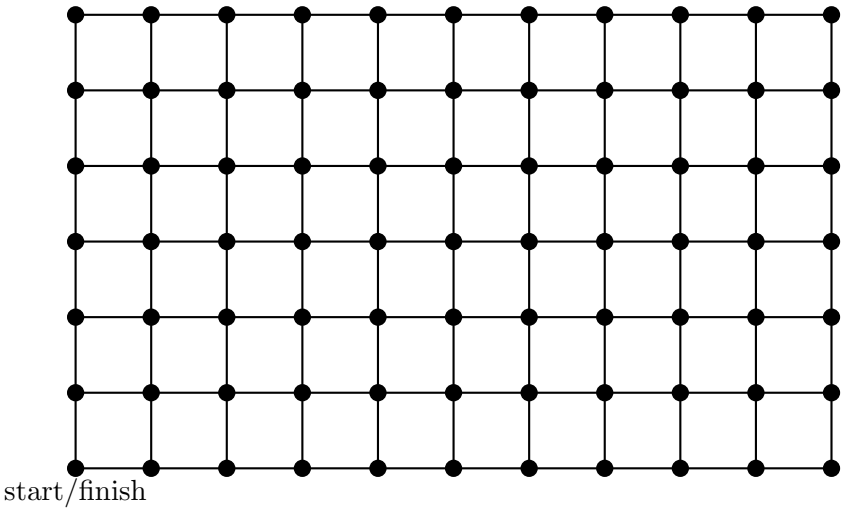
Complete **two** of the following problems.

B1. You bought a rectangular puzzle consisting of 253 pieces. Each piece is identical to one of the 5 samples shown in the diagram. Is it possible to re-assemble this puzzle? If so, how many pieces of type E are there in the puzzle? If it's not possible, explain why. You may assume that the puzzle is solvable. *Hint:* 253 is divisible by 11.



 Written by D.C. Ernst

B2. Suppose the graph below represents a city grid, where each vertex (solid dot) represents an intersection of roads and each edge (line segment joining vertices) represents a road. If you start at the vertex in the lower left corner, find a path through the city that visits every intersection exactly once and returns to where you started. If this is not possible, explain why.



B3. A mouse eats her way through a $3 \times 3 \times 3$ cube of cheese by tunneling through all of the 27 subcubes of size $1 \times 1 \times 1$ (picture a Rubik's cube). If she starts at one corner and always moves to an uneaten subcube by passing through a face of a subcube, can she finish at the center of the cube? If so, find such a path. If this is not possible, explain why.

