Quiz 2

Your Name:

Instructions

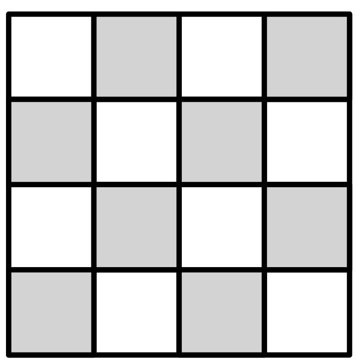
This quiz consists of two parts. In each part complete **two** problems for a total of four problems. You should provide detailed solutions on your own paper to the problems you choose to complete. I expect your solutions to contain sufficient justification. I also expect your solutions to be *well-written, neat, and organized*. Incomplete thoughts, arguments missing details, and scattered symbols and calculations are not sufficient. Each problem is worth 4 points for a total of 16 points. Good luck and have fun!

Part A

Complete **two** of the following problems.

A1. Rufus and Dufus are identical twins. They are each independently given the same 4-digit number. Rufus takes the number and converts it from decimal (base 10) to base 4, and writes down the 6-digit result. Dufus simply writes the first and last digits of the number followed by the number in its entirety. Rufus is shocked to discover that Dufus has written down exactly the same number has him. What was the original number? In other words, if the original number was $xyzw$, which number $xyzw$, when converted from decimal to base 4 becomes $xwxyzw$?

A2. Pennies and Paperclips is a two-player game played on a 4×4 checkerboard as shown below.



One player, "Penny", gets two pennies as her pieces. The other player, "Clip", gets a pile of paperclips as his pieces. Penny places her two pennies on any two different squares on the board. Once the pennies are placed, Clip attempts to cover the remainder of the board with paperclips - with each paperclip being required to cover two vertically or horizontally adjacent squares. Paperclips are not allowed to overlap. If the remainder of the board can be covered with paperclips then Clip is declared the winner. If the remainder of the board cannot be covered with paperclips then Penny is the winner.

(a) State and prove a conjecture that determines precisely every situation in which Penny wins based on the placement of the pennies.

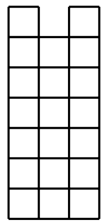
(b) State and prove a conjecture that determines precisely every situation in which Clip wins based on the placement of the pennies.

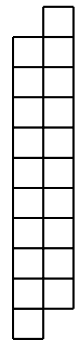
A3. How many factors of 10 are there in 50! (i.e., 50 factorial)?

Part B

Complete **two** of the following problems.

B1. Tile the following grids using every tetromino exactly once. If a tiling is not possible, explain why.

(a) 

(b) 

B2. In this problem, we will explore a modified version of the Sylver Coinage Game. In the new version of the game, a fixed positive integer $n \geq 3$ is agreed upon in advance. Then 2 players, A and B, alternately name positive integers from the set $\{1, 2, \ldots, n\}$ that are not the sum of nonnegative multiples of previously named numbers among $\{1, 2, \ldots, n\}$. The person who is forced to name 1 is the loser! Here is a sample game between A and B using the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ (i.e., $n = 10$):

(a) A opens with 4. Now neither player can name $4, 8$.

(b) B names 5. Neither player can name $4, 5, 8, 9, 10$.

(c) A names 6. Neither player can name $4, 5, 6, 8, 9, 10$.

(d) B names 3. Neither player can name $3, 4, 5, 6, 7, 8, 9, 10$.

(e) A names 2. Neither player can name $2, 3, 4, 5, 6, 7, 8, 9, 10$.

(f) B is forced to name 1 and loses.

Suppose player A always goes first. Argue that if there exists an n such that player B is guaranteed to win on the set $\{1, 2, \ldots, n\}$ as long as he or she plays intelligently, then player A is guaranteed to win on the set $\{1, 2, \ldots, n, n + 1\}$ as long as he or she plays intelligently. Your argument should describe a strategy for player A.

B3. An overfull prison has decided to terminate some prisoners. The jailer comes up with a game for selecting who gets terminated. Here is his scheme. 10 prisoners are to be lined up all facing the same direction. On the back of each prisoner's head, the jailer places either a black or a red dot. Each prisoner can only see the color of the dot for all of the prisoners in front of them and the prisoners do not know how many of each color there are. The jailer may use all black dots, or perhaps he uses 3 red and 7 black, but the prisoners do not know. The jailer tells the prisoners that if a prisoner can guess the color of the dot on the back of their head, they will live, but if they guess incorrectly, they will be terminated. The jailer will call on them in order starting at the back of the line. Before lining up the prisoners and placing the dots, the jailer allows the prisoners 5 minutes to come up with a plan that will maximize their survival. What plan can the prisoners devise that will maximize the number of prisoners that survive? Some more info: each prisoner can hear the answer of the prisoner behind them and they will know whether the prisoner behind them has lived or died. Also, each prisoner can only respond with the word "black" or "red."