Quiz 3

Your Name:

Instructions

This quiz consists of two parts. In each part complete **two** problems for a total of four problems. You should provide detailed solutions on your own paper to the problems you choose to complete. I expect your solutions to contain sufficient justification. I also expect your solutions to be *well-written, neat, and organized*. Incomplete thoughts, arguments missing details, and scattered symbols and calculations are not sufficient. Each problem is worth 4 points for a total of 16 points. Good luck and have fun!

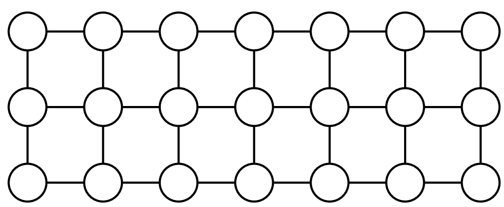
Part A

Complete **two** of the following problems.

A1. Suppose someone draws 20 distinct random lines in the plane. What is the maximum number of intersections of these lines?

A2. How many ways can 110 be written as the sum of 14 different positive integers?

A3. In the lattice below, we color 11 vertices points black. Prove that no matter which 11 are colored black, we always have a rectangle with black corners.



Part B

Complete **two** of the following problems.

B1. You need to pack several items into your shopping bag without squashing anything. The items are to be placed one on top of the other. Each item has a weight and a strength, defined as the maximum weight that can be placed above that item without it being squashed. A packing order is safe if no item in the bag is squashed, that is, if, for each item, that item's strength is at least the combined weight of what's placed above that item. For example, here are three items and a packing order:

Ordering	Item	Weight	Strength
Top	Apples	5	6
Middle	Bread	4	4
Bottom	Carrots	12	9

This packing is not safe. The bread is squashed because the weight above it, 5, is greater than its strength, 4. For homework, you found a safe packing for the 3 items above. Determine whether every collection of 3 items (not just the 3 items above) has a safe packing or provide an example that illustrates that this is not the case.

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B2. Let t_n denote the nth triangular number. Find both an *algebraic proof* and a *visual proof* of the following fact.

For all $a, b \in \mathbb{N}$, $t_{a+b} = t_a + t_b + ab$.

B3. A certain store sells a product called widgets in boxes of $7, 9$, and 11. A number n is called *widgetable* if one can buy exactly n widgets by buying some number of boxes. What is the largest non-widgetable number?

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