

Quiz 5

Your Name:

Instructions

This quiz consists of two parts. In each part complete **two** problems for a total of four problems. You should provide detailed solutions on your own paper to the problems you choose to complete. I expect your solutions to contain sufficient justification. I also expect your solutions to be *well-written, neat, and organized*. Incomplete thoughts, arguments missing details, and scattered symbols and calculations are not sufficient. Each problem is worth 4 points for a total of 16 points. Good luck and have fun!

Part A

Complete **two** of the following problems.

- A1. Consider the regular hexagon $ABCDEF$. Let X be the midpoint of CD and let Y be the midpoint of DE . Let Z be the common point of AX and BY . Which polygon has larger area, ABZ or $DXZY$?
- A2. Suppose you have 12 coins, all identical in appearance and weight except for one that is either heavier or lighter than the other 11 coins. What is the minimum number of weighings one must do with a two-pan scale in order to identify the counterfeit?
- A3. Consider an equilateral triangle with side lengths of 2 units. Find an arrangement of 5 distinct points or argue that no such arrangement exists such that all 5 points are in the interior of the triangle and every pair of points is at least 1 unit apart.

Part B

Complete **two** of the following problems.

- B1. A town of Smurfs consists of 24 blue, 8 pink, and 16 purple individuals. When two Smurfs of different colors shake hands, they both change their colors to the third color. Is it possible that all Smurfs in the town eventually have the same color? If so, describe a pattern of handshakes that will convert all the Smurfs to the same color. If this is not possible, provide a detailed argument as to why.
- B2. Consider the following dialogue.

William: I have three children.

Harry: What are their ages?

William: The product of their ages is 36.

Harry: I still don't know their ages.

William: The sum of their ages is your apartment number.

Harry: I still don't know their ages!

William: The oldest plays football.

Harry: Now I know their ages.

What are the ages of William's children?

B3. A frog jumps along the number line. It starts at 0 and every second it jumps n units (the same positive integer n each time). In addition, the frog is allowed to start by going either to the left or to the right; once it chooses a direction, it always jumps n units in that direction. We want to catch the frog. It's dark, we can't see the frog, and we do not know what n is. For all we know, it might be a super-frog, so n could be arbitrarily large. However, at any given second, we are allowed to choose an integer and search there. If the frog is on that integer, we catch it; if not, we have to try again. Describe a method for catching the frog.