Problem 73 (The Martian Artifacts). Recent archaeological work on Mars discovered a site containing a pile of white spheres, each about the size of a tennis ball. A plaque near the mound states that each sphere contains a jewel that come in many different colors while strictly more than half of the spheres contain jewels of the same color. When two spheres are brought together, they both glow white if their internal jewels are the same color; otherwise, no glow. In how few tests can you find a sphere that you are certain holds a jewel of the majority color if the number of spheres in the pile is $2, 3, 4, 5, 6, 7, 8,$ or 9? You should provide an answer with justification for each of the different values. Can you make a conjecture about how many tests are required if the number of spheres in the pile is $n \geq 2$?

Let's consider $n = 7$. There at @ least 4 majority.

$\Downarrow$

@ most 3 non-majority

3 initial tests:

①②   ③④   ⑤⑥

4 cases :

1. None of 3 pairs glow.

   Then there are at least 3 non-majority spheres (one in each pair). Then ⑦ must be majority. (3 tests)

2. Exactly one pair glows. Let's say that ①② glows while other 2 pairs do not glow. There must be @ least 2 non-majority in ③④ , ⑤⑥. Since there are @ most 3 non-majority, it

must be the case that ① and ② are majority. 3 tests

3. Exactly two pairs glow. Let's say ①② and ③④ glow while other pair does not. Since there are at most 3 non-majority, at least one pair of the glowing spheres must consist of majority color. we need another test:

②③

(a) If this pair glows, then all of ①,②,③,④ are all same color, and hence majority.

(b) If this pair does not glow, then we know that ①② have a diff color than ③④. Since there are at most 3 non-maj, we know exactly one of the pairs ①②

or ③④ consists of majority and the other consists of non-majority. Since ⑤⑥ did not glow, at least one of them is non-maj. So, we know there are at least 3 non-maj. Thus, it must be the case that ⑦ is maj.

(4 tests)

4. All 3 pairs glow. Since there at most 3 non-maj, we know at least 2 of these pairs consists of majority color. We need another test:

②③

(a) If this pair glows, then all of ①,②,③,④ have same color, and hence all are majority.

(b) If this pair does not glow, then we know that ①② and ③④ are different colors. This implies that there are at least 2 non-maj among the 7 spheres. But since ⑤⑥ glowed, it must be the case that ⑤ and ⑥ are majority.

4 tests

It would seem that we need at most 4 tests.