# Problem 3.31

(a) $\mathcal{P}(S \cap T) \subseteq \mathcal{P}(S) \cap \mathcal{P}(T)$

True

Pf: Let $S$ and $T$ be sets. Let $A \in \mathcal{P}(S \cap T)$. Then $A \subseteq S \cap T$ by def of power set. This implies that $A \subseteq S$ and $A \subseteq T$ by def of subset and intersection. But then $A \in \mathcal{P}(S)$ and $A \in \mathcal{P}(T)$ by def of power set. Thus, $A \in \mathcal{P}(S) \cap \mathcal{P}(T)$ by def of intersection. Therefore, $\mathcal{P}(S \cap T) \subseteq \mathcal{P}(S) \cap \mathcal{P}(T)$. ∎

Thm 3.42: Let $\{A_\alpha\}_{\alpha\in\Delta}$ be a collection of sets.

(a) $$\left(\bigcup_{\alpha\in\Delta} A_\alpha\right)^c = \bigcap_{\alpha\in\Delta} A_\alpha^c.$$

Pf: Let $\{A_\alpha\}_{\alpha\in\Delta}$ be collection of sets.

($\subseteq$) Let $x \in \left(\bigcup_{\alpha\in\Delta} A_\alpha\right)^c$. Then $x \notin \bigcup_{\alpha\in\Delta} A_\alpha$. This implies that $x \notin A_\alpha$ for all $\alpha\in\Delta$ (by logical DeMorgan's). But then $x \in A_\alpha^c$ for all $\alpha\in\Delta$. Hence $x \in \bigcap_{\alpha\in\Delta} A_\alpha^c$, and so $\left(\bigcup_\alpha A_\alpha\right)^c \subseteq \bigcap_\alpha A_\alpha$.

($\supseteq$) "Unzip" above. Note: Use of logical DeMorgan's shifts a sentence.