Sec 4.4: WOP

Def: max vs min

Thm 4.36: If $A \subseteq \mathbb{R}$ s.t. max (resp min) of A exists, then max (resp min) of A is unique.

Pf: Let $A \subseteq \mathbb{R}$ and assume a min of A exists. S'pose m_1 and m_2 are mins of A. Then $m_1, m_2 \in A$ and $m_1 \leq a$ and $m_2 \leq a$ $\forall a \in A$. But then $m_1 \leq m_2$ since $m_2 \in A$ and $m_2 \leq m_1$ since $m_1 \in A$. It follows that $m_1 = m_2$, and so min of A is unique. Similar pf for max. $\blacksquare$

Prob 4.37:

(a) $\max(\{5, 11, 17, 42, 103\}) = 103$

$\min(\quad " \quad) = 5$

(b) $\max(\mathbb{N}) = \text{DNE}$

$\min(\mathbb{N}) = 1$

(c) $\max(\mathbb{Z}) = \text{DNE} = \min(\mathbb{Z})$

(d) $\max((0,1]) = 1$

$\min((0,1]) = \text{DNE}$

(e) $\max((0,1] \cap \mathbb{Q}) = 1$

$\min((0,1] \cap \mathbb{Q}) = \text{DNE}$

(f) $\max((0,\infty)) = \text{DNE}$

$\min((0,\infty)) = \text{DNE}$

(g) $\max(\{42\}) = 42$

$\min(\{42\}) = 42$

(h) $\max(\{\frac{1}{n} \mid n \in \mathbb{N}\}) = 1$

$\min(\{\frac{1}{n} \mid n \in \mathbb{N}\}) = \text{DNE}$

(j) $\max(\emptyset) = \text{DNE}$

$\min(\emptyset) = \text{DNE}$

Thm 4.38 (WOP): Every nonempty subset of natural #'s has a least elmt (i.e., min)

$\emptyset \neq A \subseteq \mathbb{N} \Rightarrow \min(A)$ exists

Pf: See take-home exam.

Hint: For sake of a contradiction, assume $\emptyset \neq S \subseteq \mathbb{N}$ s.t. S does not have a min.

Goal: Prove $S = \emptyset$ using induction.

Base case: Is $1 \in S$?