

## Sec 6.1: FTA

### Def 6.1

(a) factor

(b) prime

(c) composite

Prob 6.2: 1 is neither prime nor composite

$$\mathbb{N} = \{1\} \cup \{\text{primes}\} \cup \{\text{composites}\}$$

↑ all pairwise disjoint

Prob 6.3: 2, 3, 5, 7, 11, 13, 17, 19, 23, 27, ...

Probs 6.4, 6.5: skip

Thm 6.6: If  $n \in \mathbb{N}$  w/ ~~not~~  $n > 1$ , then  $n$  can be expressed as a product of primes:

$$n = p_1 \cdots p_k,$$

each  $p_i$  is prime (not necessarily distinct).

2

Pf: See take-home exam

Hint: Let  $S = \{n \in \mathbb{N} \mid n \text{ cannot be written as prod of primes}\}$   $n > 1$  and

Goal: Show  $S = \emptyset$ .

For sake of a contradiction, assume  $S \neq \emptyset$ .

By WOP,  $S$  contains a least elmt, say  $n$ .

$n$  cannot be prime  $\Rightarrow n$  ~~composite~~ has a pair of divisors, say  $a, b$  w/  $1 \neq a, b \neq n$ .

That is,  $n = ab$ . By Thm 2.56,  $a, b < n$ .

Are  $a, b \in S$ ?

---

Where are we headed?

Goal: Thm 6.17 (FTA): Every nat  $\neq$  greater than 1 can be expressed uniquely (up to the order in which they appear) as the prod of primes

This is Thm 6.6 w/ uniqueness!

To get there, we need

• Thm 6.7 (Division Algorithm) proved for you in book

• Thm 6.13: (special case of Bezout's Lemma)

If  $p, a \in \mathbb{Z}$  s.t.  $p$  prime and  $p$  and  $a$  relatively prime, then  $\exists s, t \in \mathbb{Z}$  s.t.  $ps + at = 1$ .

• Thm 6.15 (Euclid's Lemma) Assume  $p$  prime.

If  $p \text{ divs } ab$  w/  $a, b \in \mathbb{N}$ , then either  $p \text{ divs } a$  or  $p \text{ divs } b$ .