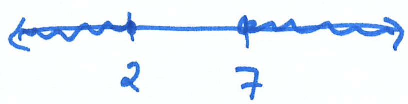
Sec 3.1

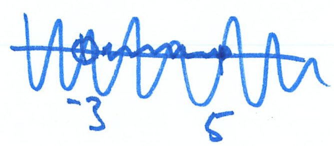
Def 3.1: set, elmts, empty set, notation

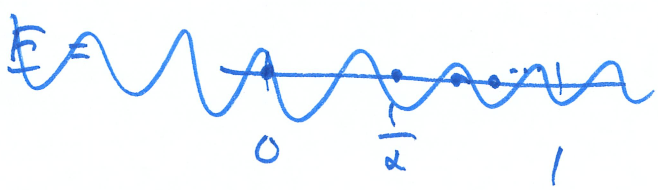
- set-builder notation
- $\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}$

3.2

(a) A = natural #'s that are mults of 3 $= \{3, 6, 9, \ldots\}$

(b) B = 

(c) C =  $\{-1, 0, 1\}$

(d)  $D = \{-2, -1, 0, 1, \ldots, 5\}$

3.3

(a) $\{t \in \mathbb{R} \mid t < -\sqrt{2}\}$

(b) $\{t \in \mathbb{R} \mid -12 < t \leq 42\}$

(c) $\{n \in \mathbb{Z} \mid n = 2k \text{ for some } k \in \mathbb{Z}\} = \{2k \mid k \in \mathbb{Z}\}$

Def 3.4: Intervals

3.5

(a) $(0,1]$

(b) $\mathbb{Z}$

Def 3.6: subset

3.7

$A = \{1,2,3\}$

$\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, A$

Thm 3.8 Discuss

3.9 $A \subseteq B$ means $(\forall x \in A)(x \in B)$ or

$(\forall x \in U)(x \in A \Rightarrow x \in B)$

Skeleton:

Let $x \in A$.

$\vdots$

Thus, $x \in B$.

Thm 3.10:

Pf: Assume A, B, C sets s.t. $A \subseteq B$ and $B \subseteq C$.

If $A = \emptyset$, then $A \subseteq C$ by thm 3.8

Let $x \in A$. Since $A \subseteq B$, $x \in B$. Since $B \subseteq C$, $x \in C$.

Thus, $A \subseteq C$.

$\square$

Def 3.11 Equality of sets

Ex. $\{1, 2, 3\} = \{1, 3, 2\}$

Thm 3.12

Pf S'pose A and B sets.

($\Rightarrow$) Assume $A = B$. Then it is clear that $A \subseteq B$ and $B \subseteq C$.

($\Leftarrow$) Now, assume $A \subseteq B$ and $B \subseteq C$. Then it is clear that A and B contain ~~say~~ same elmts, and hence $A = B$.

$\square$

Def 3.13 Proper subset ...

Def 3.14: union, intersection, set diff, comp

universe is needed

Def 3.15: Disjoint

3.16 $U = \{1, \dots, 10\}$, $A = \{1, \dots, 5\}$, $B = \{1, 3, 5\}$

(a) $A \cap C = \{2, 4\}$ $\qquad C = \{2, 4, 6, 8\}$

(b) $B \cap C = \phi$

(c) $A \cup B = \{1, 2, 3, 4, 5\} = A$

(d) $A \setminus B = \{2, 4\}$

(e) $B \setminus A = \{\ \} = \phi$

(f) $C \setminus B = \{2, 4, 6, 8\} = C$

(g) $B^c = \{2, 4, 6, \dots, 10\}$

(h) $A^c = \{6, \dots, 10\}$

(i) $(A \cup B)^c = \{1, \dots, 5\}^c = \{6, \dots, 10\}$

(j) $A^c \cap B^c$

$= \{6, \dots, 10\} \cap \{2, 4, 6, \dots, 10\}$

$= \{6, \dots, 10\}$

3.17 : Skip

3.18

(a) $S \cap T = \{x\}$

(b) $(S \cup T)^c = \{x, y, z, \{y\}\}^c = \{\{x, z\}\}$

(c) $T \setminus S = \{\{y\}\}$

Thm 3.19

Pf: Assume A, B be sets s.t. $A \subseteq B$. Let $x \in B^c$.

Then $x \notin B$. Since $A \subseteq B$, $x \notin A$. But then $x \in A^c$.

So, $B^c \subseteq A^c$. $\blacksquare$

Thm 3.20

Pf: Let A, B be sets.

($\subseteq$) Let $x \in A \setminus B$. Then $x \in A$ and $x \notin B$.

Since $x \notin B$, $x \in B^c$. Since $x \in A$ and $x \in B^c$,

$x \in A \cap B^c$. So, $A \setminus B \subseteq A \cap B^c$.

($\supseteq$) Let $x \in A \cap B^c$. Then $x \in A$ and $x \in B^c$. Since $x \in B^c$, $x \notin B$. Since $x \in A$ and $x \notin B$, $x \in A \setminus B$. So, $A \cap B^c \subseteq A \setminus B$.

Therefore, $A \setminus B = A \cap B^c$. $\blacksquare$

__Alternate:__ We see that

$$\begin{aligned} A \setminus B &= \{x \in U \mid x \in A \text{ and } x \notin B\} \\ &= \{x \in U \mid x \in A \text{ and } x \in B^c\} \\ &= A \cap B^c. \end{aligned}$$

This is __not__ always possible!

__Thm 3.21:__

(a) __Pf:__ Assume A, B are sets.

($\subseteq$) Let $x \in (A \cup B)^c$. Then $x \notin A \cup B$. By logic version of DeMorgan's, $x \notin A$ and $x \notin B$. But then $x \in A^c$ and $x \in B^c$, and so $x \in A^c \cap B^c$. Thus, $(A \cup B)^c \subseteq A^c \cap B^c$.

($\supseteq$) Let $x \in A^c \cap B^c$. Then $x \in A^c$ and $x \in B^c$. This implies that $x \notin A$ and $x \notin B$. By logic version of De Morgan's, we get $x \notin A \cup B$, and hence $x \in (A \cup B)^c$. Thus, $A^c \cap B^c \subseteq (A \cup B)^c$.

Therefore, $(A \cup B)^c = A^c \cap B^c$. $\boxed{\equiv}$

Thm 3.22:

(a) Proof: Let A, B, C be sets.

($\subseteq$) Let $x \in A \cup (B \cap C)$. Then $x \in A$ or $x \in B \cap C$.

Case 1: Assume $x \in A$. Then $x \in A \cup B$ and $x \in A \cup C$. This implies that $x \in (A \cup B) \cap (A \cup C)$.

Case 2: Assume $x \in B \cap C$. Then $x \in B$ and $x \in C$. This implies that $x \in A \cup B$ and $x \in A \cup C$.

Thus, $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$.

(2) Now, let $x \in (A \cup B) \cap (A \cup C)$. Then $x \in A \cup B$ and $x \in A \cup C$.

<u>Case 1</u>: Suppose $x \in A$. Then $x \in A \cup (B \cap C)$.

<u>Case 2</u>: O/w, suppose $x \notin A$. But then since $x \in A \cup B$ and $x \in A \cup C$, it must be the case that $x \in B$ and $x \in C$. Hence $x \in B \cap C$, which implies that $x \in A \cup (B \cap C)$.

In either case, $x \in A \cup (B \cap C)$. Therefore, $(A \cup B) \cap (A \cap C) \subseteq A \cup (B \cap C)$.

Thus, $A \cup (B \cap C) = (A \cup B) \cap (A \cap C)$. $\square$

Sec 3.3

Def 3.27: Power set...

P 3.28

(b) $B = \{a, \{a\}\}$

$\mathcal{P}(B) = \{\phi, \{a\}, \{\{a\}\}, B\}$

(c) $C = \phi$

$\mathcal{P}(C) = \{\phi\}$

(d) $D = \{\phi\}$

$\mathcal{P}(D) = \{\phi, \{\phi\}\}$

P 3.29

Claim: If $|S| = n$, then $|\mathcal{P}(S)| = 2^n$.