<u>Axiom of Induction</u>. Let $S \subseteq \mathbb{N}$ s.t.

(a) $1 \in S$, and

(b) If $k \in S$, then $k+1 \in S$.

Then $S = \mathbb{N}$.

<u>Thm 4.2 (PMI)</u>. Let $P(1), P(2), \ldots$ be a sequence of statements. Assume

(a) $P(1)$ true, and

(b) If $P(k)$ is true, then $P(k+1)$ is true,

Then $P(n)$ true for all $n \in \mathbb{N}$.

<u>Pf.</u> Let $P(1), P(2), \ldots$ be a sequence of statements. Define Oops...

$$S = \{k \in \mathbb{N} \mid P(k) \text{ is true}\}$$

"truth set".

Assume
(a) $P(1)$ true,
(b) $P(k)$ true $\Rightarrow$ $P(k+1)$ true

Since $P(1)$ true, $1 \in S$.

S'pose $k \in S$. Then $P(k)$ true by def of S.

But then $P(k+1)$ also true by hypothesis. Thus, $k+1 \in S$, as well, again by def of S.

We have satisfied ~~by~~ both conditions of hypothesis for Axiom of Ind. It follows that $S = \mathbb{N}$. Therefore, $P(n)$ is true for all $n \in \mathbb{N}$ by def of S. $\blacksquare$