

An Inquiry-Based Approach to Abstract Algebra

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This book is intended to be a task sequence for an undergraduate abstract algebra course that utilizes an inquiry-based learning (IBL) approach. You can find the most up-to-date version of these notes on GitHub:

<http://dcernst.github.io/IBL-AbstractAlgebra/>

I would be thrilled if you used these notes and improved them. If you make any modifications, you can either make a pull request on GitHub or submit the improvements via email. You are also welcome to fork the source and modify the notes for your purposes as long as you maintain the license below.

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