

## Exam 2 (Part 2)

**Your Name:**

**Names of Any Collaborators:**

### Instructions

Answer each of the following questions by beginning of class on **Wednesday, November 17**. This part of Exam 2 is worth a total of 16 points and is worth 50% of your overall score on Exam 2. Your overall score on Exam 2 is worth 20% of your overall grade.

I expect your solutions to be *well-written, neat, and organized*. Do not turn in rough drafts. What you turn in should be the “polished” version of potentially several drafts. Feel free to type up your final version. The  $\text{\LaTeX}$  source file of this exam is also available if you are interested in typing up your solutions using  $\text{\LaTeX}$ . I’ll gladly help you do this if you’d like.

Reviewing material from previous courses and looking up definitions and theorems you may have forgotten is fair game. However, when it comes to completing the following problems, you should *not* look to resources outside the context of this course for help. That is, you should not be consulting the web, other texts, other faculty, or students outside of our course in an attempt to find solutions to the problems you are assigned. This includes Chegg and Course Hero. On the other hand, you may use each other, the textbook, me, and your own intuition. Further information:

1. You may freely use any theorems that we have discussed in class, but you should make it clear where you are using a previous result and which result you are using. For example, if a sentence in your proof follows from Problem 3.16, then you should say so.
2. Unless you prove them, you cannot use any results from the course notes that we have not yet covered.
3. You are **NOT** allowed to consult external sources when working on the exam. This includes people outside of the class, other textbooks, and online resources.
4. You are **NOT** allowed to copy someone else’s work.
5. You are **NOT** allowed to let someone else copy your work.
6. You are allowed to discuss the problems with each other and critique each other’s work.

**I will vigorously pursue anyone suspected of breaking these rules.**

You should **turn in this cover page** and all of the work that you have decided to submit. **Please write your solutions and proofs on your own paper.** To convince me that you have read and understand the instructions, sign in the box below.

**Signature:**

Good luck and have fun!

When proving a result from the book, you may utilize any result that comes before it in the book (even if it is a result that you have not proved yet).

1. (4 points) Complete one of the following.
  - (a) Prove that if  $f$  is continuous on  $[a, b]$  and there exists  $c \in (a, b)$  such that  $f(c) > 0$ , then there exists an open interval  $I$  containing  $c$  such that  $f(x) > 0$  for all  $x \in I$ .
  - (b) Prove that if  $f : A \rightarrow \mathbb{R}$  is a continuous real function such that  $c \in [a, b] \subseteq A$ , then the set  $\{x \in [a, b] \mid f(x) = f(c)\}$  is closed.
  - (c) Suppose  $f : A \rightarrow \mathbb{R}$  and  $g : A \rightarrow \mathbb{R}$  are both real functions that are continuous at  $a \in A$ . Prove that if  $h : A \rightarrow \mathbb{R}$  is a real function such that  $f(a) = h(a) = g(a)$  and for all  $x \in A$ ,  $f(x) \leq h(x) \leq g(x)$ , then  $h$  is also continuous at  $a$ .
2. (4 points) Complete one of the following.
  - (a) Problem 7.12
  - (b) Problem 7.14
3. (4 points) Complete one of the following.
  - (a) Problem 8.15
  - (b) Problem 8.17
  - (c) Problem 8.18
4. (4 points) Complete one of the following.
  - (a) Problem 8.19
  - (b) Problem 8.20
  - (c) Problem 8.22