Homework 9

Combinatorics of Genome Rearrangements

You are allowed and encouraged to work together on homework. Yet, each student is expected to turn in his or her own work. In general, late homework will not be accepted. However, you are allowed to turn in **up to two late homework assignments with no questions asked**.

Reviewing material from previous courses and looking up definitions and theorems you may have forgotten is fair game. However, when it comes to completing assignments for this course, you should *not* look to resources outside the context of this course for help. That is, you should not be consulting the web, other texts, other faculty, or students outside of our course in an attempt to find solutions to the problems you are assigned. This includes Chegg and Course Hero. On the other hand, you may use each other, the textbook, me, and your own intuition. **If you feel you need additional resources, please come talk to me and we will come up with an appropriate plan of action.** Please read NAU's Academic Integrity Policy.

Complete the following problems.

1. Assume $n = 2k$ for $k \geq 2$ (so that n is even and greater than or equal to 4). Prove that $d_{bt}(n \cdots 321) = k + 1$. Consider using the ideas from Homework 8. In light of this problem and the results of Homework 8, we now know that for $n \geq 3$ (for n odd or even),

$$d_{bt}(n \cdots 321) = \left\lceil \frac{n+1}{2} \right\rceil.$$

   Feel free to dig into the *Sorting a bridge hand* paper for further insights. I had previously provided a link to this paper in Discord.

2. Define $d_{bt}^{\max}(S_n) := \max\{d_{bt}(\pi) \mid \pi \in S_n\}$. That is, $d_{bt}^{\max}(S_n)$ is the diameter of the corresponding sorting graph (and corresponding Cayley diagram). Prove

$$\left\lceil \frac{n+1}{2} \right\rceil \leq d_{bt}^{\max}(S_n) \leq \frac{3n}{4}.$$

   Hint: For the second inequality, utilize a result from class that involved $\mathrm{bp}(\pi)$.

3. Assume n is even and let $\gamma_n = (1, 3, 5, \ldots, n-1, 2, 4, 6, \ldots, n)$ (notice that we have used cycle notation, not one-line notation). Discover as much as you can about $d_{bt}(\gamma_n)$. Perhaps you will be able to determine the actual value, but at least aim for lower and upper bounds.